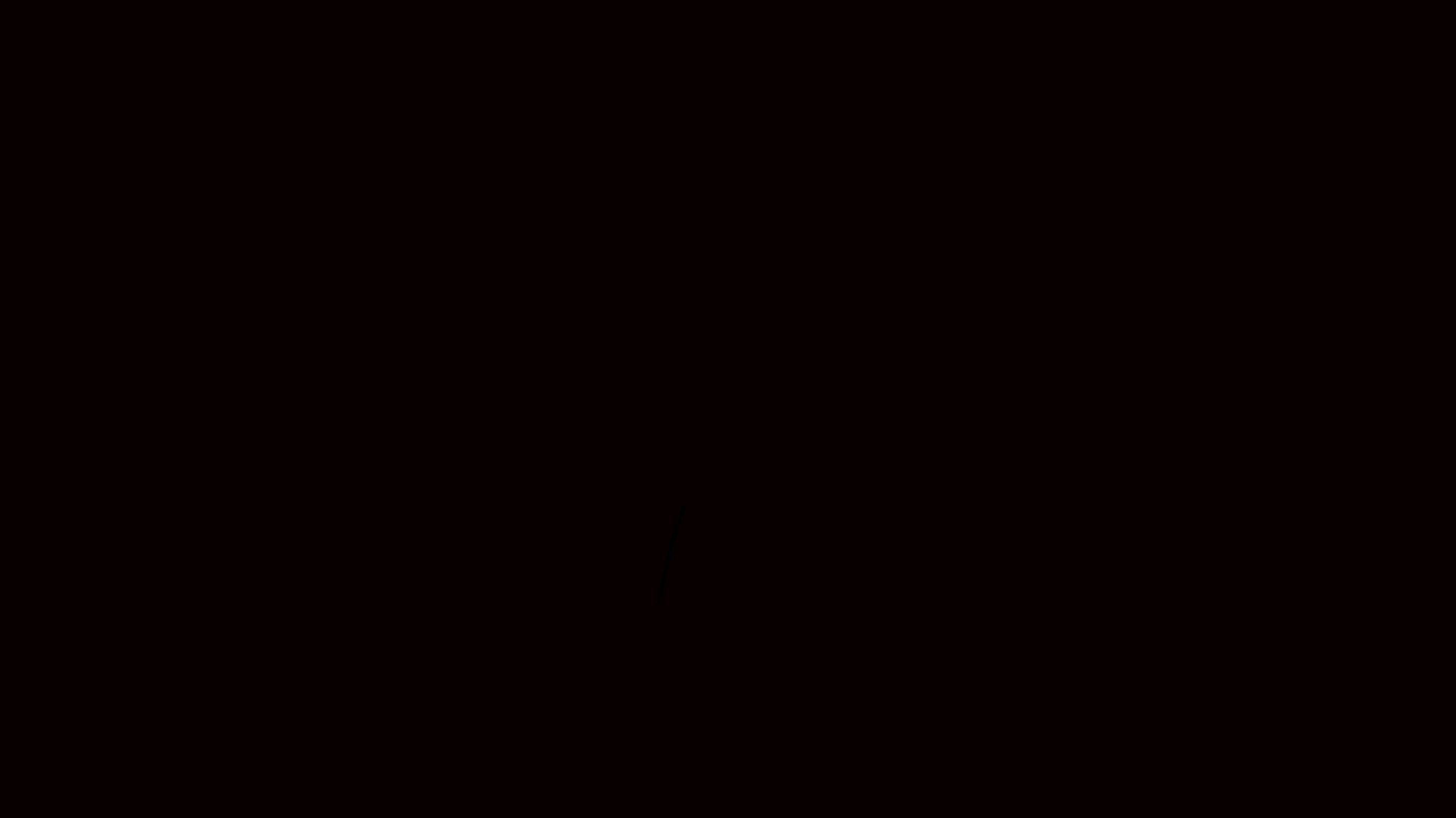


Three Dimensional Geometry

D'Ratio and D'Cosine of the vector.

Line

Directed line

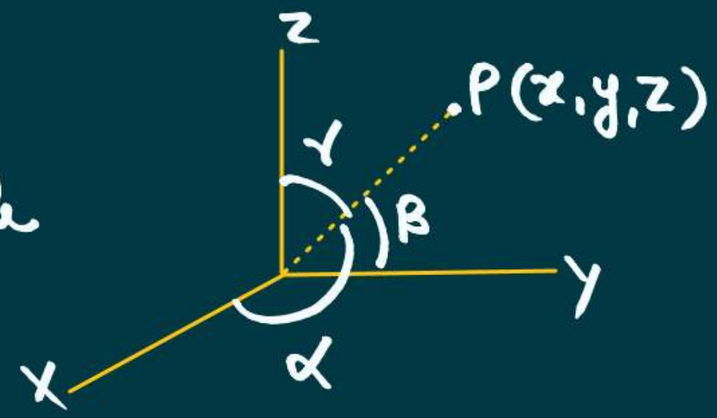
$\overrightarrow{OP} = \vec{r}$

$\alpha, \beta, \gamma \rightarrow$ Direction angle

$\cos\alpha, \cos\beta, \cos\gamma \rightarrow$ Direction Cosine

Direction Ratio $= l, m, n$

Directed line Segment of vector.

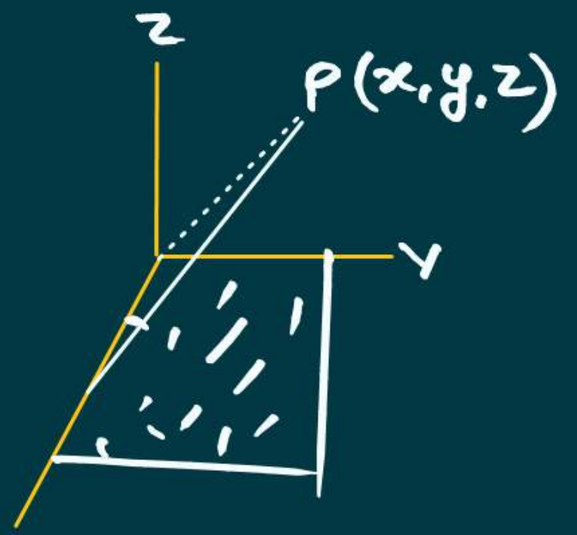


Number proportional of D'cosine.

$$\frac{a}{l} = \frac{b}{m} = \frac{c}{n} = \lambda$$

$$a = \lambda l \quad m = b\lambda \quad c = n\lambda$$

a, b, c are D'R Number ∄ D' Ratio.



x, y, z – D' Ratio

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

x, y, z → Direction Ratios.

$$l^2 + m^2 + n^2 = 1$$

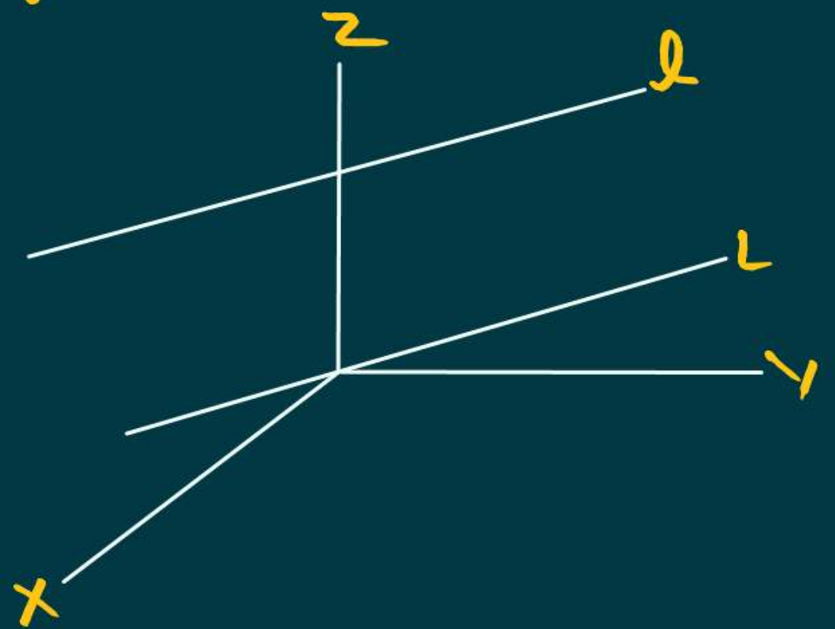
Direction Cosine & Direction Ratio of line

l = Directed line

L = Directed line $\parallel$ to l passing through Origin.

α, β, γ are D' Angle.

$\cos\alpha, \cos\beta, \cos\gamma$ are D' Cosine of line.



$$l^2 + m^2 + n^2 = 1$$

Direction Ratio of Line.

$$l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}}$$

$$m = \pm \frac{b}{\sqrt{a^2+b^2+c^2}}$$

$$n = \pm \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Direction cosine of line two point.

$P(x_1, y_1, z_1)$ $Q(x_2, y_2, z_2)$

$$\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}$$

$\vec{a} \perp \vec{b}$

$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

$\vec{a} \parallel \vec{b}$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Ex 2.1

1. i) Find the distance of the point P(2,-3,-4) From the x-axis.

$Q(2,0,0)$ $P(2,-3,-4)$

$$PQ = \sqrt{(2-2)^2 + (-3-0)^2 + (-4-0)^2}$$

$$= \sqrt{0+9+16}$$

$$= \sqrt{25} = 5 \text{ unit}$$

ii) Find the distance of the point Q(5,-4,12) From the y-axis.

$P(0,-4,0)$ $Q(5,-4,12)$

$$PQ = \sqrt{(5-0)^2 + (-4+4)^2 + (12-0)^2}$$

$$= \sqrt{25+144}$$

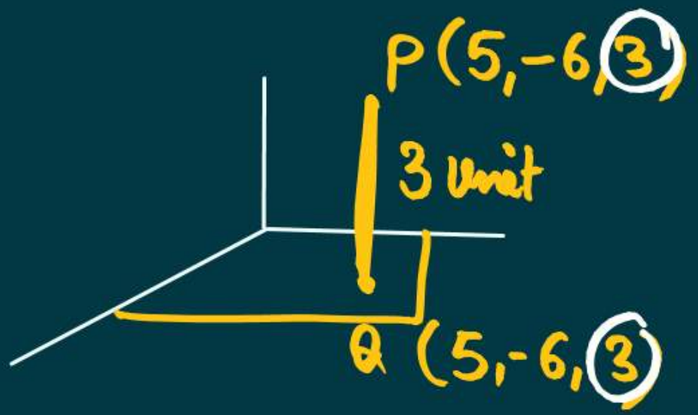
$$= \sqrt{169} = 13 \text{ unit.}$$

iii) Find the distance of the point P(5,-6,3) From the XOY Plane.

Q (5,-6,0) P(5,-6,3)

$$PQ = \sqrt{(5-5)^2 + (-6+6)^2 + (3-0)^2}$$

$$= 3$$



iii) Find the distance of the point Q(2,-4,6) From the ZOX Plane.

$P(2,0,6)$ $Q(2,-4,6)$

$$PQ = \sqrt{(2-2)^2+(-4-0)^2+(6-6)^2}$$

$$PQ = \sqrt{16} = 4 \text{ unit}$$

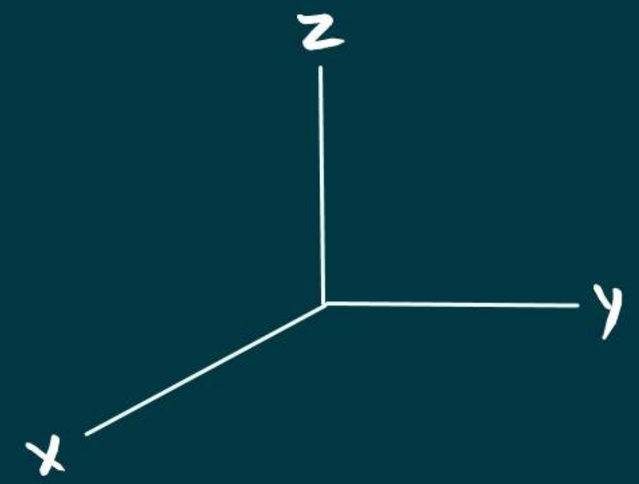
2.i) if a line makes angles $90^0, 60^0$ *and* 30^0 with the positive direction of x, y *and* $z-$ *axes respectively* find its direction cosines.

ii) Write the direction cosines of z-axis $z=0$

$\alpha = 90^\circ$, $\beta = 60^\circ$, $\gamma = 30^\circ$

$\cos 90^\circ = 0$ $\cos 60^\circ = \frac{1}{2}$ $\cos 30^\circ = \frac{\sqrt{3}}{2}$

(ii)



Direction cosine.

$\cos 90^\circ \quad \cos 90^\circ \quad \cos 0^\circ$

$(0 \quad , \quad 0 \quad , \quad 1)$

3. i) Can the numbers $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ be the direction cosines of a line

$$l^2 + m^2 + n^2 = 1$$

$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} \neq 1$$

It is not D' Cosine of line.

ii) Can a line have directions angles $45^0, 60^0, 120^0$?

iii) Prove that 1,1,1 cannot be direction cosines of a straight line.

4. If the direction cosines of a line are $<\frac{1}{c}, \frac{1}{c}, \frac{1}{c}>$ then find the value of c